Introduction

Welcome to Math 125

1. Introduction to Calculus
2. PreCalculus Review

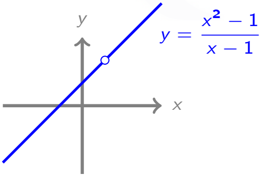
by Joseph Phillip Brennan
Jila Niknejad

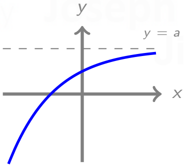
Introduction to Calculus

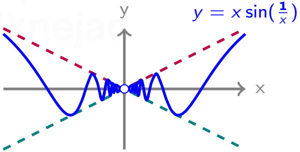
Calculus is the study of **change**. That is,
(1) study of change when elapsed time is extremely small (infinitesimal).
(2) calculating the behavior of mathematical models.

Limits and Continuity (Chapter 2)

Limits and continuity are essential in deep understanding of the infinitesimal change and finding the behavior of models.

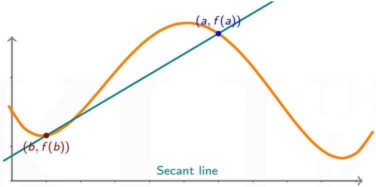




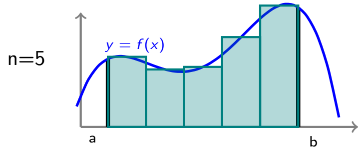


Some Concepts in Calculus I

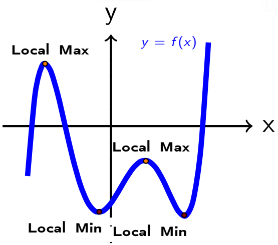
Rate of Change (Chapter 3)



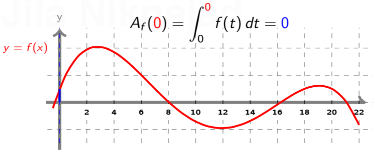
Area Under Curves (Chapter 5)



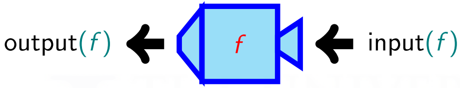
Optimization (Chapter 4)



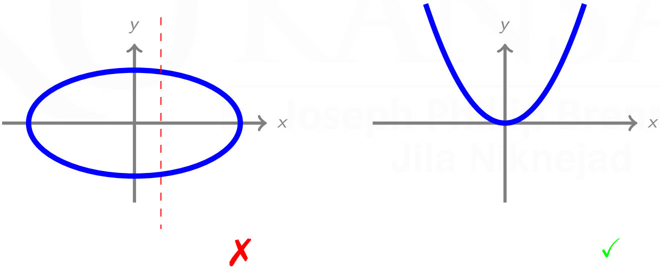
Fundamental Theorem



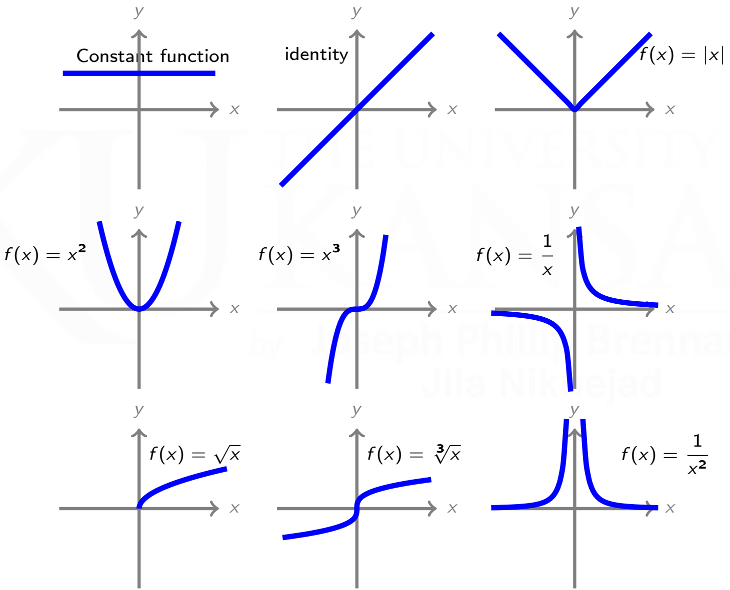
Function as a Black Box



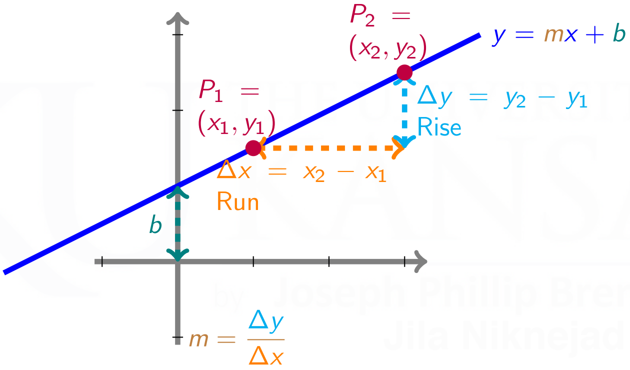
Vertical Line Test



Graphs of Well-known Functions

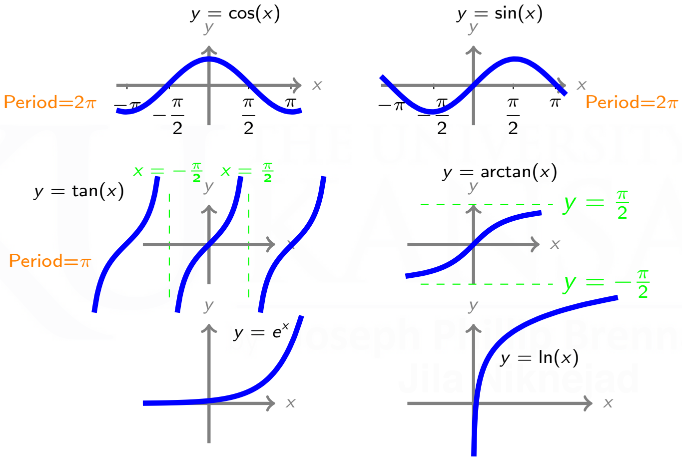


Equation of a Line



- $y = mx + b$ is called the **slope intercept form.** $y = \underbrace{m}_{\text{Slope}} x + \underbrace{b}_{\text{y-intercept}}$
- Graph of a linear function is a line.

Trig, Exponential and Logarithmic Graphs



What are the Horizontal and Vertical Asymptotes of these graphs? What are the symmetries? How are the graphs of inverse functions related?

Example, Domain and Composition of Functions

- Composition and domain $f(x) = \sqrt{x}, g(x) = \dfrac{x+1}{x-2}$.

Laws of Exponents and Logarithms

Laws of Exponents

Let a and b be positive numbers and let x and y be real numbers. Then,

1. $b^x . b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Laws of Logarithms

If m and n are positive numbers and $b > 0$, $b \neq 1$, then

1. $\log_b(mn) = \log_b(m) + \log_b(n)$
2. $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$
3. $\log_b(m^n) = n \log_b(m)$
4. $\log_b(1) = 0$
5. $\log_b(b) = 1$

Examples, Laws of Exponents and Logarithms

- Evaluate: **(1)** $\log_3\left(\frac{1}{27}\right)$ (1) $\log_5(125)$

- Solve: **(1)** $\ln(x) + \ln(x-1) = 1$ **(2)** $e^{7-4x} = 6$

Examples

- Simplify: $\frac{x^{-2}y\sqrt{z^5}}{x\sqrt{y^4z}}$

- Factoring and Simplification

$$\text{Factor: } 18(x+2)^{-2}(x-1)^{\frac{1}{2}} - 6(x+2)^{-1}(x-1)^{\frac{3}{2}}$$

Graph of Functions and Horizontal Asymptotes

Horizontal asymptotes indicate the stable end behavior of functions. Many elementary functions that we are familiar with have horizontal asymptotes and a review of all of those are necessary.

Example:

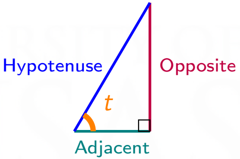
Graph the following function indicating its asymptotes.

$y = -e^{-x+1} + 1$

$y = \arctan(x)$

Trigonometric Ratios

Hypotenuse is the side opposite of the right angle.
Adjacent is the other side of the angle x.
Opposite is the side opposite of angle x.



$$\sin(t) = \frac{\textit{opposite}}{\textit{hypotenuse}} \qquad \tan(t) = \frac{\textit{opposite}}{\textit{adjacent}} \qquad \sec(t) = \frac{\textit{hypotenuse}}{\textit{adjacent}}$$

$$\cos(t) = \frac{\textit{adjacent}}{\textit{hypotenuse}} \qquad \cot(t) = \frac{\textit{adjacent}}{\textit{opposite}} \qquad \csc(t) = \frac{\textit{hypotenuse}}{\textit{opposite}}$$

Examples, Trigonometric Ratios

- $\frac{\sin(x)}{x} \neq \sin \qquad \sin(x+y) \neq \sin(x) + \sin(y)$
- Find other trig functions if $\sin(\theta) = \dfrac{3}{5}, 0 < \theta < \dfrac{\pi}{2}$.

Example, Piecewise-defined

Graph

$$f(x) = \begin{cases} x & x < -2 \\ 0.5x & -2 \leq x \leq 0 \\ -2x & 0 < x < 4 \\ 3x - 16 & x \geq 4 \end{cases}.$$

Label two points of each linear piece of the graph.

Absolute Value Functions

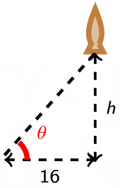
- Absolute Value functions are piecewise-defined functions.

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

- Rewrite the absolute value function as a piecewise-defined function:
  $f(x) = |x - 2|$

Examples, Modeling with Mathematics

An observer is viewing the space shuttle take off from a distance of 16 kilometers from the launch pad. The shuttle travels straight up with constant speed of 8 kilometers per second for the first part of the ascent. Let t be the time passed, in seconds, since the shuttle has been launched. Let θ be the angle of elevation in radians.



(1) Express the the vertical distance of the shuttle, h, as a a function of time, t, in seconds.

(2) Express the angle of elevation θ as a function of the vertical distance of the shuttle, h, in kilometers.

(3) Express the angle of elevation, θ, as a function of time, t, in seconds.

(4) What is the vertical distance, in kilometers, when $t = 2$ seconds.

(5) What is the angle of elevation, in radians, when $t = 2$ seconds.

Examples, Modeling, Optimization

What is the **maximum area** of a rectangle inscribed in a right triangle with side lengths 3 and 4, if the sides of the rectangle are parallel to the legs of the triangle?